

Problem 7

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.185185\dots$$

Solution

$$\begin{aligned} 0.185185\dots &= 0.185 + 0.000185 + 0.000000185 + \dots \\ &= \frac{185}{1000} + \frac{185}{1\,000\,000} + \frac{185}{1\,000\,000\,000} + \dots \\ &= \sum_{i=1}^{\infty} \frac{185}{10^{3i}} \\ &= \sum_{i=1}^{\infty} \frac{185}{1000^i} \\ &= 185 \sum_{i=1}^{\infty} \frac{1}{1000^i} \\ &= 185 \sum_{i=1}^{\infty} \left(\frac{1}{1000}\right)^i \\ &= 185 \left[-1 + \sum_{i=0}^{\infty} \left(\frac{1}{1000}\right)^i \right] \\ &= 185 \left[-1 + \frac{1}{1 - \left(\frac{1}{1000}\right)} \right] \\ &= \frac{5}{27} \end{aligned}$$